UNIT 1

FOCUS A

THE LONG ROAD TO CALCULUS

The origins of calculus go back over 2000 years to the work of the Greeks on areas and tangents. Archimedes (287-212 B.C.) found the area of a section of a parabola, an accomplishment that amounts in our terms to evaluating $\int_0^b x^2 dx.$ He also found the area of an ellipse and both the surface area and the volume of a sphere. Apollonius (around 260-200 B.C.) wrote about tangents to ellipses, parabolas, and hyperbolas, and Archimedes discussed the tangents to a certain spiral-shaped curve. Little did they suspect that the “area” and “tangent” problems were to converge many centuries later. With the collapse of the Greek world, which had survived for a thousand years, it was the Arab world that preserved the works of Greek mathematicians. In its liberal atmosphere, Arab, Christian, and Jewish scholars worked together, translating and commenting on the old writings, occasionally adding their own embellishments.

It was not until the seventeenth century that several ideas came together to form calculus. In 1637, both Descartes (1596–1650) and Fermat (1601–1665) introduced analytic geometry. Descartes examined a given curve with the aid of algebra, while Fermat took the opposite tack, exploring the geometry hidden in a given equation. In this same period, Cavalieri (1598–1647) found the area under the curve $y = x^n$ for $n = 1, 2, 3, \dots, 9$ by a method the length of whose computations grew rapidly as the exponent increased. Stopping at $n = 9$, he conjectured that the pattern would continue for larger exponents. “What about the other exponents?” we may wonder. Before 1665 there were no other exponents. Nevertheless, it was possible to work with the function which we denote $y = x^{p/q}$ for positive integers p and q by describing it as the function y such that $y^q = x^p$. Wallis (1616–1703) found the area under the curve by a method that smacks more of magic than of mathematics. However, Fermat obtained the same result with the aid of an infinite geometric series.

The problem of determining tangents to curves was also in vogue in the first half of the seventeenth century. Descartes showed how to find a line perpendicular to a curve at a point P (by constructing a circle that meets the curve only at P); the tangent was then the line through P perpendicular to that line. Fermat found tangents in a way similar to ours and applied it to maximum-minimum problems. The stage was set for the union of the “tangent” and “area” techniques. Indeed, Barrow (1630–1677), Newton’s teacher at Cambridge, obtained a result equivalent to the fundamental theorem of calculus, but it was not expressed in a useful form.

Newton (1642–1727) arrived in Cambridge in 1661, and during the two years 1665–1666, which he spent at his family’s farm to avoid the plague, he developed the essential calculus – recognizing that finding tangents and calculating areas are inverse processes. The first integral table ever compiled is to be found in one of his manuscripts of this period. He also introduced negative and fractional exponents, thus demonstrating that such diverse operations as multiplying a number by itself several times, taking its reciprocal, and finding a root of some power of that number are just special cases of a single general exponential function a^x, where x is a positive integer, –1, or a fraction. Independently, however, Leibniz (1646–1716) also invented calculus. A lawyer, diplomat, and philosopher, for whom mathematics was a serious avocation, Leibnitz established his version in the years 1673–1676. To Leibnitz we owe the notation *dx* and *dy*, the terms “differential calculus” and “integral calculus”, the integral sign, and the word “function”.

It was to take two more centuries before calculus reached its present state of precision and rigor. The notion of a function gradually evolved from “curve” to “formula” to any rule that assigns one quantity to another. The great calculus text of Euler, published in 1748, emphasized the function concept by including not even one graph. In several texts of the 1820s, Cauchy (1789–1857) defined “limit” and “continuous function” much as we do today.

He also gave a definition of the definite integral, which with a slight change by Riemann (1826–1866) in 1854 became the definition standard today. So by the mid-nineteenth century the discoveries of Newton and Leibnitz were put on a solid foundation. In 1833, Liouville (1809–1882) demonstrated that the fundamental theorem could not be used to evaluate integrals of all elementary functions. In fact, he showed that the only values of the constant k for which $\int \sqrt{1-x^2}\ \sqrt{1-kx^2}\,dx$ is elementary are 0 and 1.

Still some basic questions remained, such as "What do we mean by area?" (For instance, does the set of points situated within some square and having both coordinates rational have an area? If so, what is this area?) It was as recently as 1887 that Peano (1858-1932) gave a precise definition of area – that quantity which earlier mathematicians had treated as intuitively given.

(From: Stein S. K.: *Calculus and Analytic Geometry,* pp. 218–219 - adapted)

Notice:

- the pronunciation and the stress of the words:
  infinite [’infinət] *definite* [’definət] *finite* [’fainait] *infinity* [in’finəti]
- the difference in meanings of the words *hypothesis* and *conjecture*:
  hypothesis [hai’poθisis] *(n)* – an idea or explanation for something based on known facts but not yet proved (*čes. hypotéza, předpoklad) – srov.: hypothesize* [hai’poθisaiz] *(v), hypothetical* [,haipəu’θetikəl] *(adj)*
  conjecture [kən’džekčə] (*n, v*) – a conclusion or opinion based on incomplete evidence, on the appearance of a situation rather than on proof *(*čes. *dohad, domněnka) – srov.: conjectural* [kən’džekčərəl] (adj)
- the difference in meanings of the words *ellipse* and *ellipsis*:
  ellipse (*pl.* ellipses) [i’lips/~iz] *(n)* – a term from geometry (*čes.* elipsa)
  ellipsis (*pl.* ellipses) [i’lipsiz/~i:z] *(n)* – a term from style, a series of dots that usually indicate an intentional omission of a word, sentence or whole section from the original text being quoted, an ellipsis can also be used to indicate an unfinished thought (…) (*čes.* v jazykovém a polygrafickém kontextu, elipsa, výpustka)
- inversion in "*<u>Little did they suspect</u> that the "area" and "tangent" problems were to converge many centuries later.*" (about inversion – see Unit 6)

Exercises

1. ***In the passage above find all the nouns which form their plurals irregularly and those that have only a singular or plural form. (Write their singular and plural forms if possible).***

2. ***Put into the singular form:***
   a) The experiments may confirm the hypotheses.
   b) Symmetric polyhedra are two polyhedra each of which is congruent to the mirror image of the other.
   c) Abscissae are horizontal coordinates in two-dimensional systems of rectangular coordinates.
   d) Rhombi are parallelograms with adjacent sides equal.
   e) DNA is stored in the nuclei of cells.
   f) The radii of these wheels are 30 cm.
   g) Students are writing their theses on black holes.
   h) The points where the axes (1) cut the ellipse are the vertices (2).
   i) Is the German language on the curricula at British schools?

3. Choose the correct word for each of the following sentences and give the plural form. Each word may be used only once:

(analysis, apex, crisis, criterion, formula, lemma, locus, maximum, medium, phenomenon)

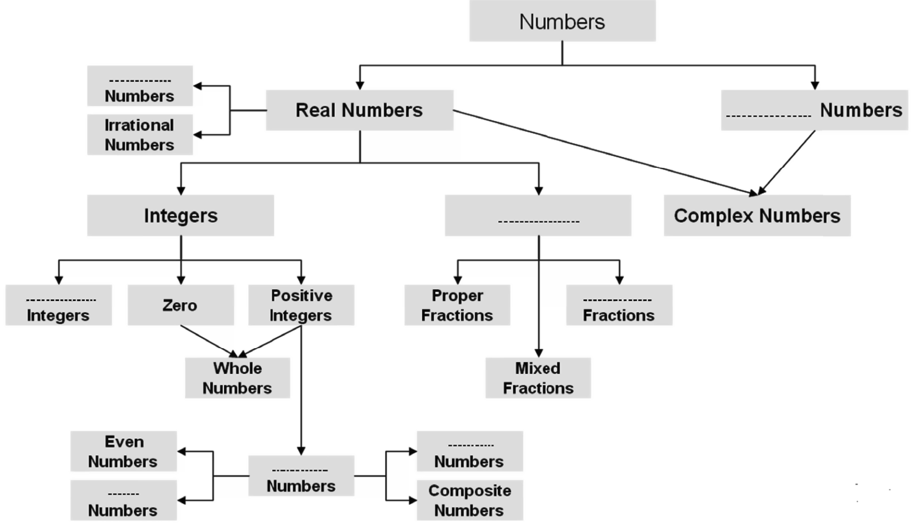
a) ……… are the highest points relative to some lines or planes.
b) ……… are observed events.
c) ……… are decisive moments.
d) ……… are channels of communication.
e) ……… are theorems which are proved to be used in the proof of other theorems.
f) ……… are standards or means of judging.
g) ……… are general expressions for solving problems.
h) ……… are separations of things into their parts or components.
i) ……… of a set (as defined in set theory) are the greatest values in the set.
j) In geometry, ………..are collections of points which share properties.

4. Fill in a suitable expression in the correct form. Each form may be used only once:

(crisis, criterion, datum, emphasis, focus, formula, matrix, nucleus, radius, spectrum)

a) Much ……………….has been given to the careful exposition of details.
b) We had to learn many chemical ……………. at school but I can only remember H_2O for water.
c) The ……………… was/were collected by various researchers.
d) The Health Service should not be judged by financial ……………. alone.
e) All the line segments extending from the centre of a circle are called …………
f) In the 17th century the word ……… was introduced into optics, referring to the range of colours observed when white light was dispersed through a prism.
g) The sets of colours into which beams of light can be separated are called …………
h) I've passed several ………………. during my illness, but the fever's started to go down since yesterday.
i) Nuclear fission means the dividing of a ……………. (1) and nuclear fusion means the joining of the ………………(2).
j) ……………….can only be added or subtracted if they have the same order.

5. Complete the following diagram:



State whether the following statements are true or false:

a) An integer is a rational number.
b) A rational is an integer.
c) A number is either a rational or an irrational, but not both.

Classify according to number type; some numbers may be of more than one type:

a) 0.45
b) 3.14159265358979323846264338327950288419716939937510**...**
c) 3.14159
d) 10
e) $^{5}/_{3}$
f) $1\,^{2}/_{3}$
g) –sqrt(81)
h) $^{-9}/_{3}$

6. ***Rewrite these expressions into mathematical symbols:***

a) *x* asterisk is approximately equal to point two one three
b) the modulus of *x* is less than or equal to *x* factorial
c) ten choose nine is equal to ten
d) *x* double-dashed is greater than *x* hat
e) binomial n over *k* does not equal the (*k* + 1)-th binomial coefficient of the (*n* + 1)-th degree

7. ***Read out the following notation (Fundamental Symbols and Combinatorics):***

a) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
b) $C_k(n) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
c) $a^* \geq \hat{a} \geq \tilde{a}$
d) $a \leq |a|$
e) $A = (5; +\infty)$
f) $a \approx 2.056$
g) $P(n) = n!$
h) $P'(k_1, k_2, k_3, ..., k_n) = \frac{(k_1 + k_2 + k_3 + ... + k_n)!}{k_1! \cdot k_2! \cdot k_3! \cdot ... \cdot k_n!}$
i) $|a| - |b| \leq ||a| - |b|| \leq |a \pm b| \leq |a| + |b|$
j) $V_k(n) = \frac{n!}{(n-k)!}$
k) $|a| = |-a|$
l) $|x| > 5$
m) $x \leq 5$
n) $C'_k(n) = \binom{n+k-1}{k}$

8. ***Read the assignments below. Find solutions to the problems and explain the procedures:***

a) $\binom{10}{9} = ?$

b) $\binom{5}{0} = ?$

c) $V_3(5) = ?$

d) $P(3) = ?$

e) $C_2'(8) = ?$

9. ***Prove the following equalities:***

a) $\binom{n}{k} = \binom{n}{n-k}$

b) $\binom{n}{1} = n$

c) $\binom{n}{n} = \binom{n}{0} = 1$

d) $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

e) $\binom{n}{k}\binom{k}{r} = \binom{n-r}{k-r}\binom{n}{r}$

f) $\binom{n+1}{k} = \frac{n+1}{n-k+1}\binom{n}{k}$

FOCUS B

GEOMETRY

Fundamental signs:

||	is parallel to
⊥	is perpendicular to
Δ	a triangle
∠	an angle
∡	a measured angle
∢	a spherical angle
∟	a right angle
○	a circle
(,)	coordinates
AB	a line
$\overrightarrow{AB}$	a ray
$\lvert x\text{-}y \rvert$	a distance
°	a degree of an arc
≅	is congruent to
≈	is similar to
≡	is identical with
'	a minute
"	a second